# Probability and Random Process
## Lecture 9

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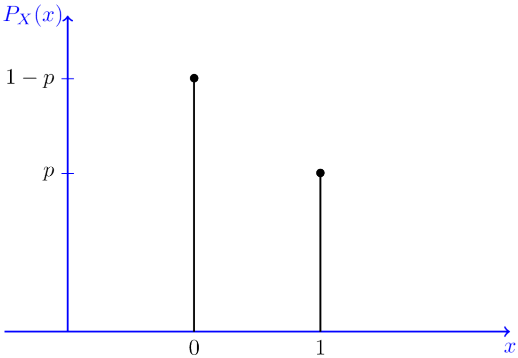
# Review

1. Bernouli trials and binomial distributions
2. Gaussian distribution
3. Poisson distribution
4. Uniform distribution

# Bernoulli distribution

A Bernoulli trial has two possible outcomes, success and failure.
The probability of success is $p$ and the probability of failure is $1 - p$ where $p \in \{0, 1\}$

$X \sim Bernoulli(p)$



# Bernoulli distribution

$E(X) = p$
$Var(X) = p(1-p)$

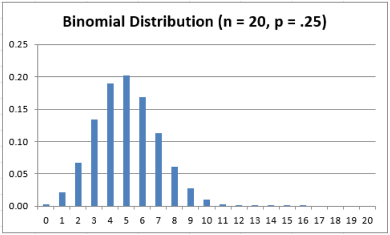
# Binomial distribution

Consider a Bernoulli trial with parameter p. The binomial distribution models the number of success in N independent Bernoulli trial.
Given n independent trials the number of successes k can vary from 0 to n
The pdf of binomial distribution is

$$f(k, n, p) = \binom{n}{k} p^k (1-p)^{n-k}$$

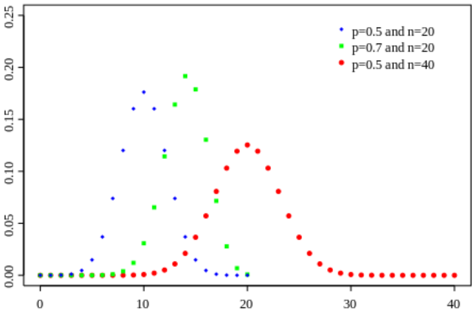
$$\sum_{0}^{n} \binom{n}{k} p^k (1-p)^{n-k} = (p + (1-p))^n = 1$$

# Binomial distribution



# Binomial distribution

If X is binomialy distributed random variable
$E(X) = np$
$var(X) = np(1 - p)$



# Gaussian distribution

The probability density function of a Gaussian distributed random variable is given by

$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(x-\mu)^2}{2\sigma^2})$

The expected value of a Gaussian random variable is $\mu$

The variance of Gaussian random variable is $\sigma$

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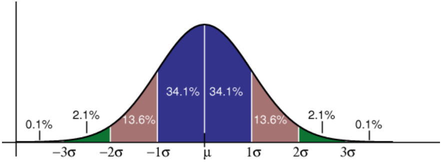
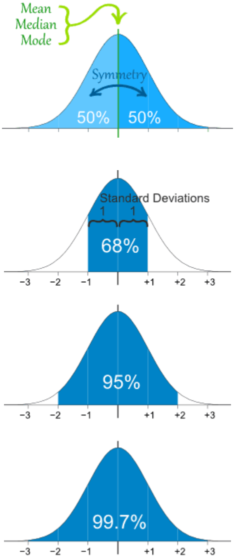


# Normal distribution

A Gaussian distribution with $\mu = 0$ and $\sigma^2 = 1$ is called a normal distribution.
$f_X(x) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{x^2}{2})$

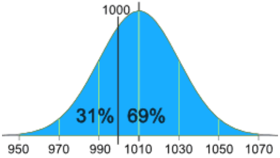
# Normal distribution



# Example of Gaussian

Your company packages sugar in 1 kg bags.
When you weigh a sample of bags you get these results:

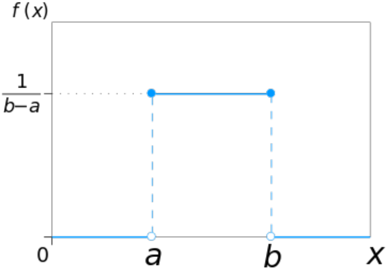
- 1007g, 1032g, 1002g, 983g, 1004g, ... (a hundred measurements)
- Mean = 1010g
- Standard Deviation = 20g




To reduce error adjust the machine so that 1000g is at -3 standard deviations away from mean.

# Uniform distribution

The probability density function of uniform distribution is
$f_X(x) = \frac{1}{b-a}$ for $a \leq x \leq b$
$f_X(x) = 0$ otherwise



# Uniform distribution

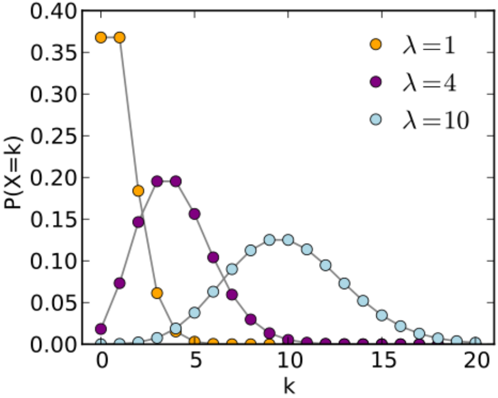
Expected value $= \frac{1}{1}(a+b)$
The variance is $\frac{1}{12}(b-a)^2$

# Poisson distribution

A discrete random variable X is said to have a Poisson distribution with parameter $\lambda > 0$ if, for $k = 0, 1, 2, ...,$
$f(k, \lambda) = P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$

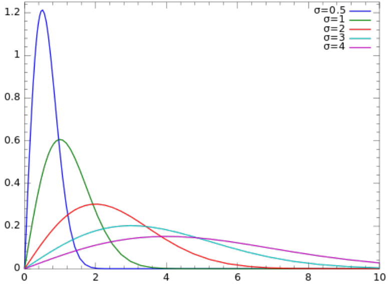
$E(X) = Var(X) = \lambda$

# Poisson distribution



# Rayleigh distribution

If $U1$ and $U2$ are independent normal variables with mean 0 and standard deviation $\sigma > 0$ then$X = \sqrt{U1 + U2}$ has the Rayleigh distribution with scale parameter $\sigma$ .
The pdf of Rayleigh distribution is given by
$f(x, \sigma) = \frac{x}{\sigma^2} e^{-\frac{x^2}{(2\sigma^2)}}, \quad x \geq 0$
The expected value is $\sigma\sqrt{\frac{\pi}{2}}$
The variance is $\frac{4-\pi}{2}\sigma^2$



# Sum of random variables

Suppose you have two random variables $X$ and $Y$ with expected values $\mu_X$ and $\mu_Y$, and variances $\sigma_X^2$ and $\sigma_Y^2$

The expected value of $X + Y$ will be $\mu_X + \mu_Y$.

The variance of $X + Y = \sigma_X^2 + \sigma_Y^2$

The same can be extended to n random variables.

# iid random variables

Independent and identically distributed random variables.
Throw the dice 5 times and observe the values. Each throw is independent of every other throw. But all the throws have identical distribution.

# Central limit theorem

Consider n iid random variables $X_i$

We take N samples from each of the above variables and compute the sample mean. Central limit theorem states that the probability distribution of sample mean will approach Gaussian distribution as $N$ approaches infinity.

# Central limit theorem:illustration

Consider four 6 faced dice. on every dice ,each face has a probability of $1/6$. The dices are independent and identically distributed with mean 3.5.
Suppose we throw the set of dice several times. For each throw we get a sample.
2 4 5 6
1 3 2 5
1 2 2 5
For each sample we can compute the mean.
The probability distribution of this mean will tend to a gaussian with mean $\mu_x$ and variance $\sigma^2/n$