Probability and Random Process
Lecture 8

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Review

1. Covariance
2. Correlation
3. Independent random variables
4. Orthogonal random variables

Covariance

Covariance is defined as
Cov(X,Y)= $E((X-\mu_X)(Y-\mu_Y))$

$$Cov(X,Y) = E(XY) - \mu_X\mu_Y$$

The correlation of two random variables X and Y is defined as
$corr(X,Y) = \frac{Cov(X,Y)}{\sigma_X\sigma_Y}$
Where σ_X and σ_Y are the standard deviations of X and Y respectively.
$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(x-\mu)}{\sigma^2})$

Independence

Two random variables are independent if
$F_{X,Y}(x, y) = F_X(x)F_Y(y)$

Two random variables are said to be orthogonal if
$E(XY) = 0$

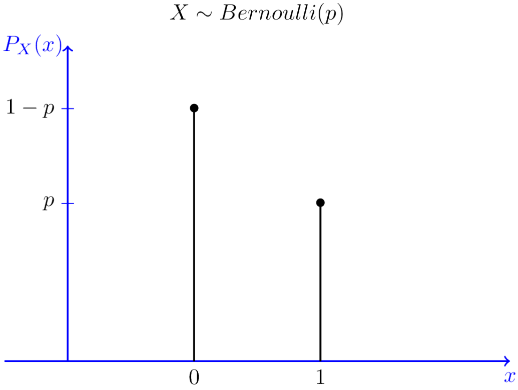
Common probability Density functions

We will look at the following density function.

1. Bernoulli Distribution
2. Binomial Distribution
3. Uniform Distribution
4. Gaussian Distribution
5. Exponential Distribution
6. Poisson Distribution
7.

Bernoulli distribution

A Bernoulli trial has two possible outcomes, success and failure.
The probability of success is p and the probability of failure is $1 - p$ where $p \in \{0, 1\}$



Bernoulli distribution

$E(X) = p$
$Var(X) = p(1 - p)$

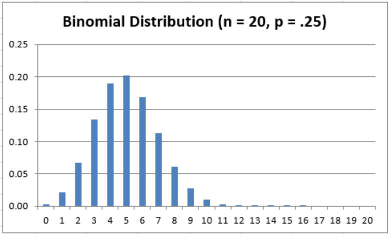
Binomial distribution

Consider a Bernoulli trial with parameter p. The binomial distribution models the number of success in N independent Bernoulli trial.
Given n independent trials the number of successes k can vary from 0 to n
The pdf of binomial distribution is

$$f(k,n,p) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\sum_{0}^{n} \binom{n}{k} p^k (1-p)^{n-k} = (p + (1-p))^n = 1$$

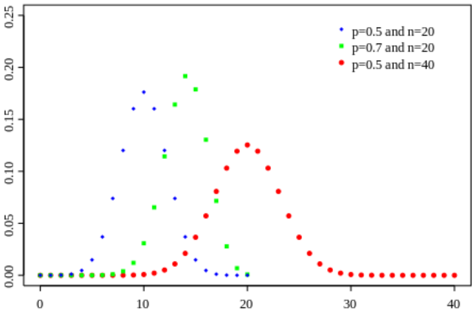
Binomial distribution



Binomial distribution

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(x-\mu)}{\sigma^2})$$

If X is binomialy distributed random variable
$E(X) = np$
$var(X) = np(1-p)$



Gaussian distribution

The probability density function of a Gaussian distributed random variable is given by

$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(x-\mu)^2}{2\sigma^2})$

The expected value of a Gaussian random variable is μ

The variance of Gaussian random variable is σ

Gaussian distribution

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$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(x-\mu)^2}{2\sigma^2})$
The expected value of a Gaussian random variable is μ
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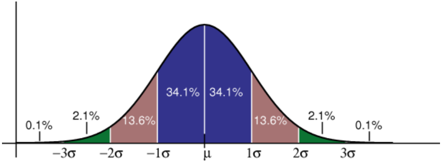
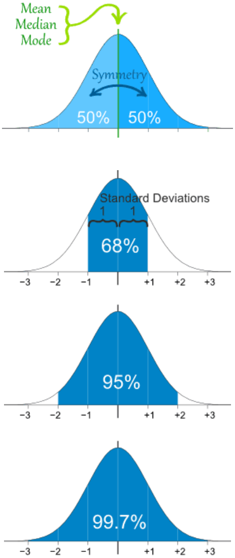


Normal distribution

A Gaussian distribution with $\mu = 0$ and $\sigma^2 = 1$ is called a normal distribution.
$f_X(x) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{x^2}{2})$

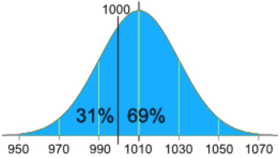
Normal distribution



Example of Gaussian

Your company packages sugar in 1 kg bags.
When you weigh a sample of bags you get these results:

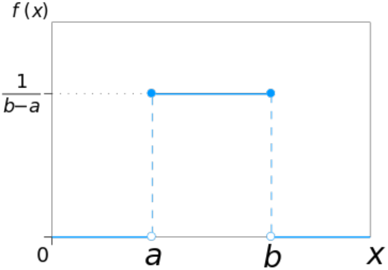
- 1007g, 1032g, 1002g, 983g, 1004g, ... (a hundred measurements)
- Mean = 1010g
- Standard Deviation = 20g



To reduce error adjust the machine so that 1000g is at -3 standard deviations away from mean.

Uniform distribution

The probability density function of uniform distribution is
$f_X(x) = \frac{1}{b-a}$ for $a \leq x \leq b$
$f_X(x) = 0$ otherwise



Uniform distribution

Expected value $= \frac{1}{1}(a+b)$
The variance is $\frac{1}{12}(b-a)^2$

Poisson distribution

A discrete random variable X is said to have a Poisson distribution with parameter $\lambda > 0$ if, for $k = 0, 1, 2, ...,$

$f(k, \lambda) = P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$

$E(X) = Var(X) = \lambda$

Poisson distribution