Probability and Random Process
Lecture 7

Sunil Thomas T

College of Engineering Attingal

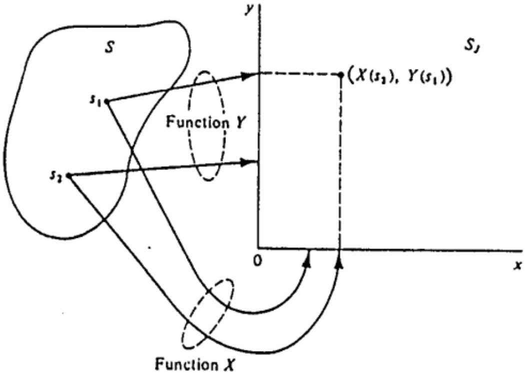
suniltt@gmail.com

August 17, 2015

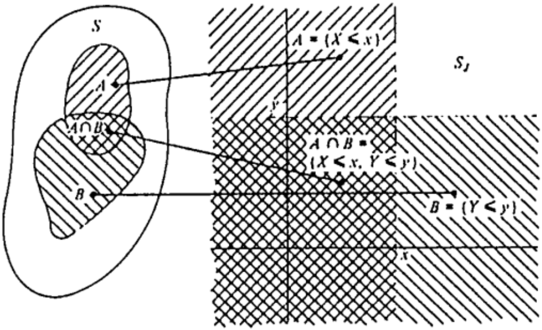
Review

1. Multiple Random variables
2. Joint Distributions
3. Marginal Distributions
4. Joint Density functions

Two random variables



Two random variables



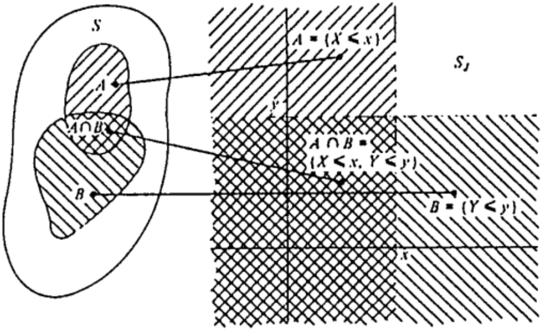
Joint Distribution

We define the event $\{X \leq x, Y \leq y\}$

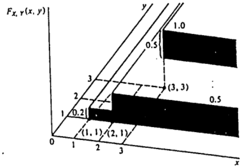
The joint probability distribution function is
$F_{X,Y}(x, y) = P\{X \leq x, Y \leq y\}$

Marginal distribution

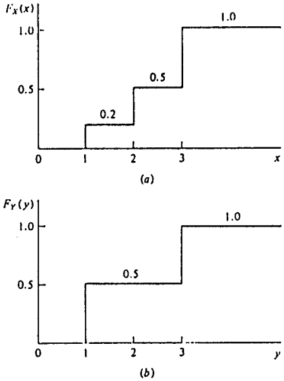
$$F_{X,Y}(x,\infty) = F_X(x) \quad F_{X,Y}(\infty, y) = F_Y(y)$$



Marginal distribution Example



(a)



(b)

Joint Density function

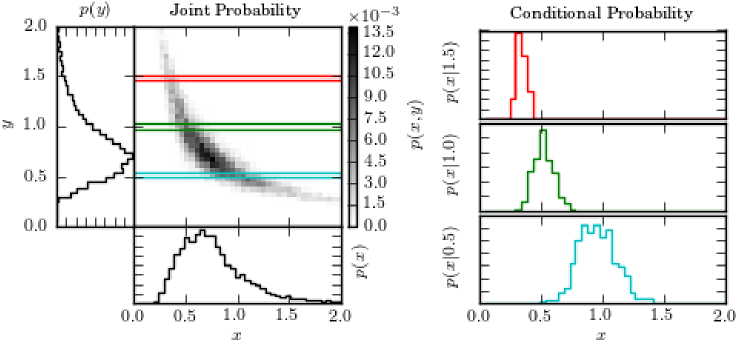
$$f_{X,Y}(x, y) = \frac{\partial^2 F_{X,Y}(x,y)}{\partial x \partial y}$$

Conditional distribution and density functions

For two continuous random variables X and Y we can define the conditional distribution as
$F_X(x|Y=y) = \frac{P(X \leq x, Y \leq y)}{P(Y \leq y)}$

Conditional density as $f_X(x|Y=y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$

Conditional distribution and density functions



Joint Moments

Just like the moments of a random variable provide a description of the random variable, the joint moments can be defined to provide description of two random variables.

For two continuous random variables X and Y the joint moment of order m+n is defined as
$E(X^m Y^n) = \int_{-\infty}^{\infty} x^m y^n f_{X,Y}(x,y) dxdy$

The joint central moment of order m+n is defined as
$E((X - \mu_X)^m (Y \mu_Y)^n) = \int_{-\infty}^{\infty} (x - \mu_X)^m (y - \mu_Y)^n f_{X,Y}(x,y) dxdy$
Where $\mu_X = E(X)$ and $\mu_Y = E(Y)$

Covariance

The most important joint moment is the covariance

Covariance is defined as
Cov(X,Y)= $E((X - \mu_X)(Y - \mu_Y))$

$$\begin{aligned} Cov(X,Y) &= E((X - \mu_X)(Y - \mu_Y)) \\ &= E(XY - \mu_Y X - \mu_X Y + \mu_X \mu_Y) \\ &= E(XY) - \mu_Y E(X) - \mu_X E(Y) + \mu_X \mu_Y \\ &= E(XY) - \mu_X \mu_Y \end{aligned}$$

Correlation

The correlation of two random variables X and Y is defined as
$corr(X, Y) = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$
Where σ_X and σ_Y are the standard deviations of X and Y respectively.

The correlation varies between 1 and -1.
Two variables are uncorrelated if correlation is 0.

Statistical Independence

Recall that two events A and B are independent if $P(A \cap B) = P(A)P(B)$
This can be extended to random variable. Consider two random variables X and Y

X and Y are said to be statistically independent if and only if
$P(X \leq x, Y \leq y) = P(X \leq x)P(Y \leq y)$
This implies the joint distribution
$F_{X,Y}(x, y) = F_X(x)F_Y(y)$
And the density
$f_{X,Y}(x, y) = f_X(x)f_Y(y)$

Independent random variables are uncorrelated.
Uncorrelated random variables need not be independent.

Orthogonal random variables

Two random variables are said to be orthogonal if
$E(XY) = 0$