Probability and Random Process
Lecture 4

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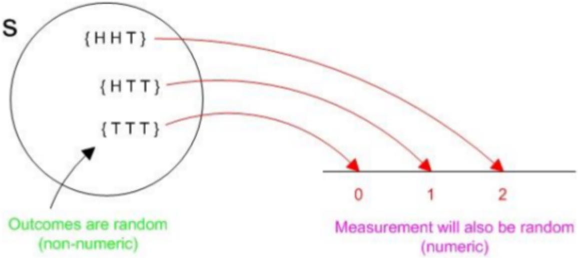
Overview

1. Review
2. Probability Density Function
3. Expected value and Variance

Review

What is a random variable.



Meaning of the notation $\{\mathbf{X} \leq x\}$

As defined earlier the random variable $\mathbf{X}$ is a function from sample space to real line.

- $\{\mathbf{X} \leq x\}$ means the set of all outcomes in sample space such that $\mathbf{X}(\mathbf{s}) \leq x$
- $\{x_i \leq \mathbf{X} \leq x_2\}$ represent an event (subset) in sample space such that $x_1 \leq \mathbf{X}(\mathbf{s}) \leq x_2$ where s belongs to this event.
- $\{\mathbf{X} = x\}$ is a subset of sample space such that $\{\mathbf{X} = x\}$

Distribution Function

We have seen that $\{\mathbf{X} \leq x\}$ is an event in the sample space.
$P(\{\mathbf{X} \leq x\})$ is the probability of the event.

It is a number between 0 and 1.

We call $P(\{\mathbf{X} \leq x\})$ as the cumulative distribution function (CDF) of the random variable $\mathbf{X}$
$F_{\mathbf{X}}(x) = P\{\mathbf{X} \leq x\}$

Probability Density Functions

The probability density function is defined as the derivative of the distribution function. [1]

$f_{\mathbf{X}}(x) = \frac{dF_{\mathbf{X}}(x)}{dx}$

[1] See Peebles for a discussion on the existence of pdf in the case of discrete random variables

Properties of Probability Density Functions

1. $0 \leq f_{\mathbf{X}}(x)$ for all x
2. $\int_{-\infty}^{\infty} f_{\mathbf{X}}(x)dx = 1$

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2. $\int_{-\infty}^{\infty} f_{\mathbf{X}}(x)dx = 1$
3. $F_{\mathbf{X}}(x) = \int_{-\infty}^{x} f_{\mathbf{X}}(y)dy$
4. $P\{x_1 < X \leq x_2\} = \int_{x_1}^{x_2} f_{\mathbf{X}}(y)dy$

Discrete random variables and probability mass function

Discrete random variables have a countable number of outcomes
$f_{\mathbf{X}}(x) = P(\mathbf{X} = x)$

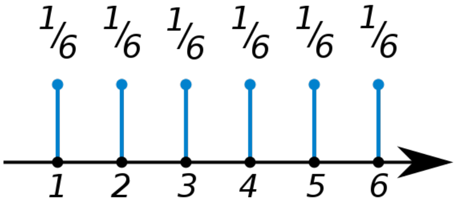


Figure: Probability mass function of a fair dice

The PMF should satisfy.

1. $0 \leq f_{\mathbf{X}}(x)$ for all x
2. $\sum_{\forall x} f_{\mathbf{X}}(x) = 1$

Discrete random variables and probability mass function

If a discrete random variable X has the following probability mass function.

x	1	2	3
P(x)	k	2k	k

Find $P(X \leq 2)$

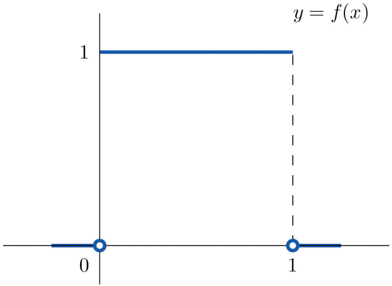
Continuous Random variables and PDF

The probability density function that accompanies a continuous random variable is a non- negative function that integrates to 1.

1. $0 \leq \mathrm{f}_{\mathbf{X}}(x)$ for all x
2. $\int_{-\infty}^{\infty} \mathrm{f}_{\mathbf{X}}(x) dx = 1$

Continuous Random variables and PDF

Example:
Uniform distribution.



$f_X(x) = 1$ for $0 \leq x \leq 1$
$\int_0^1 f_X(x)dx = \int_0^1 1dx = 1$

Expected value: Motivation:

Averages, Mean
Weighted average.

Expected value: Definition

Discrete random variable

$$E(X) = \sum_{\forall x} x f_X(x)$$

Continuous random variable

$$E(X) = \int_{-\infty}^{\infty} x f_X(x) dx$$

Alternate notations
$E(X) = \mu$
$f_X(x) = p(x)$

Variance

Variance of a random variable is $E[(X-\mu)^2]$