Probability and Random Process

Lecture 2

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Overview

Review

1. Experiments, Outcomes, Events and Sample Spaces
2. What is probability
3. Probability Space

Probability axioms

- $P(E) \geq 0$
- $P(\Omega) = 1$
- Mutually exclusive events $E_1, E_2, \ldots$ satisfies $P(\bigcup_{i=1}^{\infty} E_i) = \sum_{i=0}^{\infty} E_i$

[1] The condition is called σ additivity

Probability Space

The Triplet $\{\Omega, F, P\}$

Ω is the sample space
F sigma algebra
P Probability measure

σ-algebra

1. $\Omega \subset F$
2. F is closed under complimentation. If $A \in F$ then $A^C \in F$
3. F is closed under countable union. If $A, B \in F$ then $A \cup B \in F$

Example

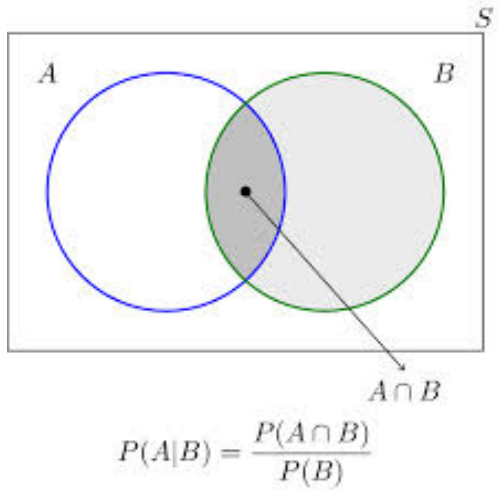
$\Omega = \{a, b, c, d\}$, a possible sigma algebra (sigma field) on Ω is
$F = \{\phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}\}$

Joint Probability

If the events A and B are not mutually exclusive the
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Conditional Probability

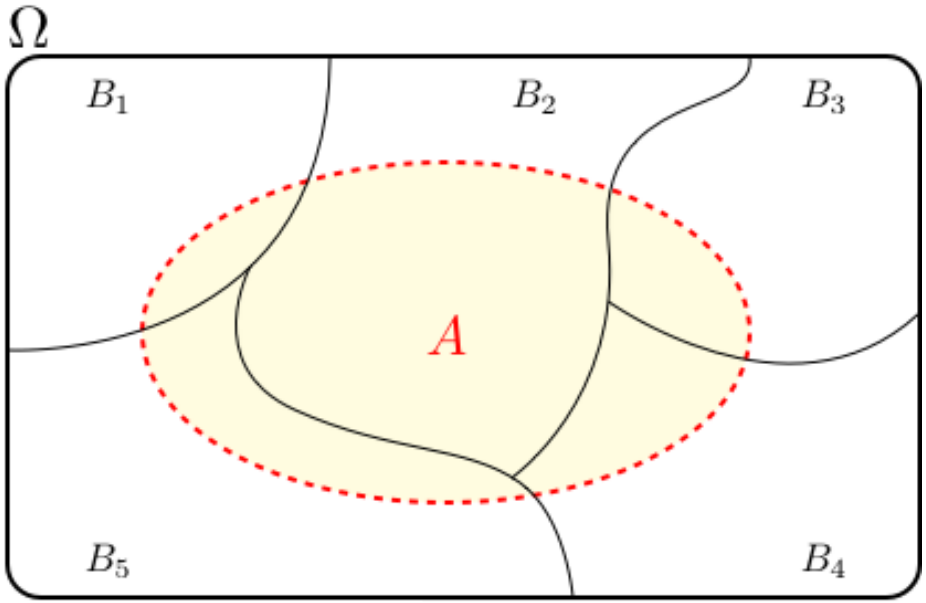
Given some event B with non zero probability.
$P(B) > 0$
Conditional probability of an event A given B
$P(A|B) = \frac{P(A \cap B)}{P(B)}$



$P(A \cap B) = P(A|B)P(B)$

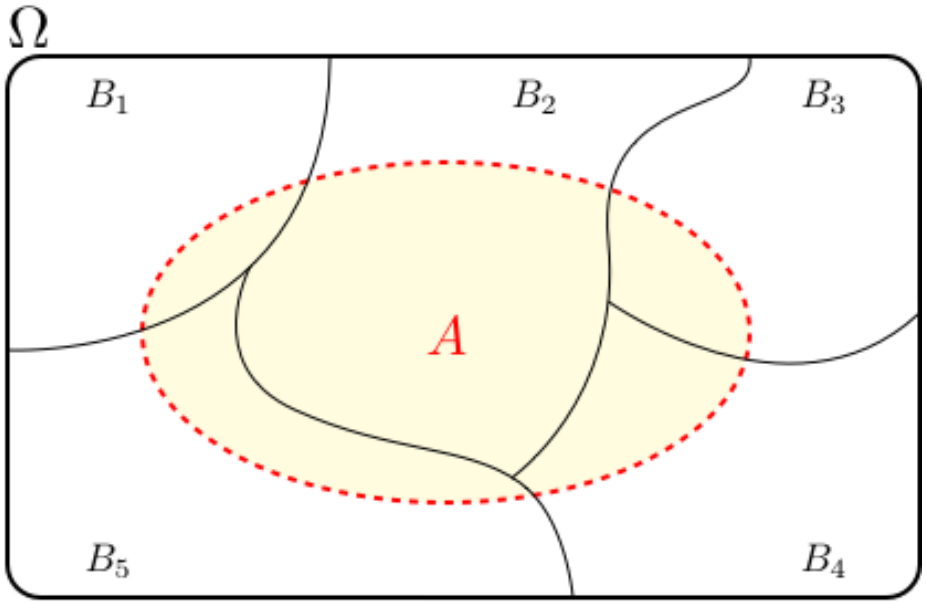
Total Probability

The probability of any event A defined on a sample space can be expressed in terms of conditional probabilities.
Suppose we have N mutually exclusive events. $B_n n = 1, 2 \cdots N$ whose union equals the sample space.

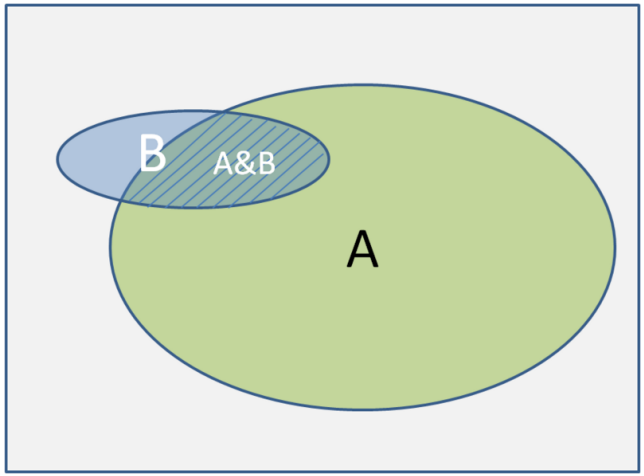


Total Probability

$$P(A) = \sum_{n=1}^{N} P(A|B_n)P(B_n)$$



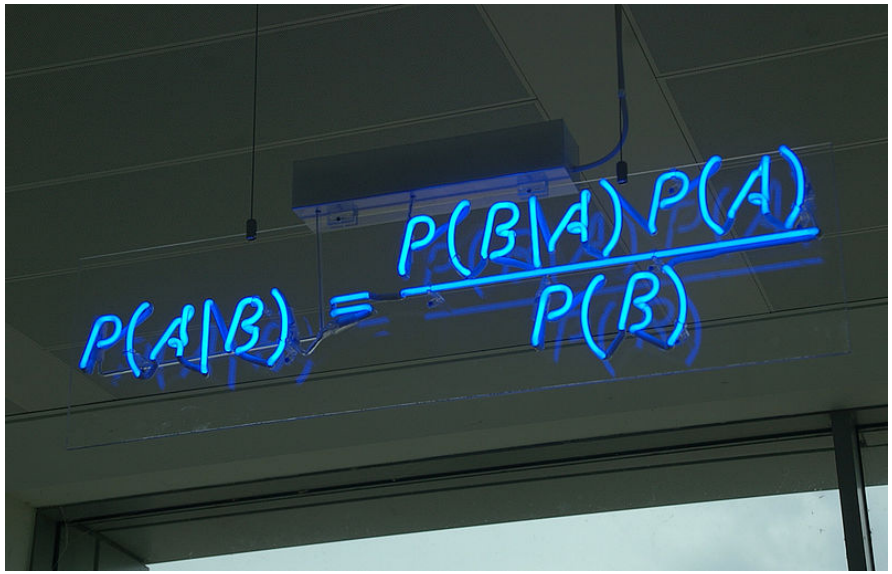
Bayes' Theorem [1]



$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$
$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$
$$P(A \cap B) = P(A|B)P(B)$$
$$P(A \cap B) = P(B|A)P(A)$$
$$P(A|B)P(B) = P(B|A)P(A)$$

[1] Read the Wikipedia page on bays' theorem

Bayes' Theorem [2]



Highly recommended. Watch these video on Bayes' Theorem.

https://youtu.be/2Df1sDAyRvQ

https://youtu.be/Zxm4Xxvzohk

[2] Image curtsey Wikipedia